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An Integer Programming Approach to Crop Rotation Planning at an Organic Farm

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Abstract

Crop rotation is an important component of organic farming because it can help to maintain healthy soil and provide weed and pest control without the use of synthetic fertilizers or pesticides. Dickinson College has a certified organic farm consisting of 24 different fields that grow 13 different crop groups. We develop a mixed-integer program that determines a four-year crop rotation schedule that takes into account crop yield requirements, irrigation type, weed control, feeding habits, and other considerations. The extensive model was constructed using an iterative process in which the model evolved to address issues identified by the farm managers at each iteration.

Introduction

Dickinson College, a small private liberal arts college in Pennsylvania, maintains a 50-acre, USDA-certified organic farm. This farm not only provides food to the campus and local community, but also serves as a “living laboratory” where students learn about sustainable agriculture through on-site classes and independent research projects. Since the college farm is certified as organic, it utilizes agricultural methods that preserve the environment, and therefore avoids synthetic fertilizers and pesticides.

A cornerstone of organic production is crop rotation, the practice of changing the type of crop grown on a particular piece of land from year to year. Crop rotation is a critical feature of organic farming because it...
provides the principal mechanism for building healthy soils, controlling pests, and minimizing diseases.

Good crop rotations require strategic planning because a lack of planning can lead to serious problems, such as the buildup of soil-borne diseases or imbalances in soil nutrients. The rotation of crops also helps to limit pest populations because it can interrupt pest life cycles by altering their habitats. The combinatorial nature of the problem, related to factors such as succession of crops, irrigation type, feeding habits, weed pressure, fallow periods, market demands, and space restrictions, makes developing a crop rotation schedule extremely complex.

We describe an optimization model that determines a four-year crop rotation schedule that takes into account crop yield requirements, irrigation type, weed control, feeding habits, and other considerations. Our work is the result of a senior student-faculty research project during the 2012–2013 academic year. Prior to our study, the college farm determined the crop rotation schedule using a manual procedure involving several spreadsheets. This time-consuming process often produced rotations that violated crop rotation principles because of the difficulty of foreseeing the future consequences of planting decisions.

Our model was created over several months in an incremental way. We began by constructing a basic model that generated a crop-rotation schedule, which we then presented to the farm managers for review. After adding variables and constraints to address issues raised by the farm managers, we repeated this process until the managers were satisfied with the results. This iterative process was utilized because the farm managers found it difficult to enumerate all the planting principles at the outset, since their own rotations were generated through intuition and experience as opposed to explicit rules.

Interestingly, since the college farm is nonprofit, our model does not focus on maximizing profit or crop yield; it focuses instead on meeting projected market demand without violating the rotation principles identified by the farm managers. We suggested to the farm managers the possibility of having the model determine the amounts and types of crops planted to maximize profit, but they were not interested in doing so. They only wanted help in figuring out how to distribute the predetermined monthly portions of the crop groups within the fields while adhering to a reasonable set of planting principles. These planting principles help to reduce labor requirements and improve the quality of the produce.

**Literature Review**

A number of authors have considered using optimization modeling to determine crop rotation schedules—see Dury et al. [2012] for a good survey. The typical objective of these models is to maximize profit or mini-
mize cost. El-Nazer and McCarl [1986] used a linear program to identify an optimum long-run crop rotation strategy with a focus on maximizing profit. Their approach does not require the modeler to explicitly predetermine the possible rotations, but rather, the model develops a plausible set of rotations based on specified principles.

Sarker et al. [1997] used linear programming to maximize the contribution from the agricultural sector of Bangladesh by determining the amount of land that should be used for each crop of a particular crop combination, while taking into consideration food demands, capital, and available land. The goal was to help encourage farmers to cultivate the most attractive crops on their land consistent with the national crop-mix plan.

Haneveld and Stegeman [2005] proposed a linear programming-based model to determine crop succession information. Utilizing information about what crop sequences are not allowed on the same piece of land, they determined suitable crop rotations over multiple years. Predetermined crop sequences that are not admissible from an expert point of view were used as constraints. Detlefsen and Jensen [2007] were also interested in developing crop sequences, but they utilized network flow modeling instead of linear programming.

dos Santos et al. [2011] developed a 0-1 optimization model for organic vegetable crop production. Their objective was to maximize the land use subject to neighborhood and succession restrictions for crops of the same botanic family. Alfandari et al. [2011] also developed a mixed-integer programming model for a multi-period crop rotation problem with demand constraints and incompatibility constraints between cultivation and fallow state on a land plot. They applied this model to a case study on Madagascar farms in the scope of a sustainable development campaign against deforestation. In particular, their main goal was modeling minimization of space consumption for crop-rotation.

While the modeling approaches mentioned above typically utilize constraints to specify crop sequences that are allowed and disallowed, our model takes a different approach. In particular, we develop a set of planting principles that need to be maintained, and then we allow the model to determine which sequences are desirable.

Fundamentals of the Farm

In 2013, the Dickinson College farm consisted of 24 fields. A map of the layout is provided in Figure 1. Crops are grown in beds (or growing strips), which are organized into rows. The width of the beds is typically determined by the type of equipment used on the farm (for example, a walk-behind tractor has a width of 30 inches). The amount of space available for planting in each field is measured in bed feet. The total amount of bed feet available in each of the fields is listed in Table 1.
The farm produces more than 50 types of fruits and vegetables, organized into 13 different crop groups for the purpose of determining the crop rotation. As a separate step not under the purview of this study, the farm forecasts the number of bed feet needed for each crop group during the planting season, which is broken into six different periods:

1. March and April (Mar/Apr)
2. May (May)
3. June (June)
4. July (July)
5. August (Aug)
6. September and October (Sept/Oct)
The estimated market demand (in bed feet) of the 13 crop groups during the six different planting periods is provided in Table 2. The farm managers determined these market demands by estimating the needs of the campus Dining Hall along with those of the local farmers’ market and the college’s Campus Supported Agriculture (CSA) program, which is a produce subscription service. Since the farm is nonprofit, the market demands attempt to balance revenue projections with the desire to supply a large variety of produce.

Our primary goal is to develop a crop rotation schedule that specifies the quantity, location, and time period for each crop group’s planting, so as to meet the market demand as closely as possible without violating any planting principles. Because planting decisions have consequences that can last as long as three years, we use a four-year planning horizon, which also can account for “end-effects,” the distortions introduced by employing a finite planning horizon. We assume that market demands are constant throughout the four-year rotation.

Although the model produces a four-year rotation, the model should be re-optimized yearly to take into account any changes, such as updated market demands or modifications to the available bed feet due to field conditions.

### Table 2.

<table>
<thead>
<tr>
<th>Crop Number</th>
<th>Crop Group</th>
<th>Mar/Apr</th>
<th>May</th>
<th>June</th>
<th>July</th>
<th>Aug</th>
<th>Sep/Oct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Brassica</td>
<td>365</td>
<td>0</td>
<td>200</td>
<td>1,475</td>
<td>1,975</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Carrot</td>
<td>433</td>
<td>767</td>
<td>1,500</td>
<td>1,900</td>
<td>533</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Cucurbits</td>
<td>0</td>
<td>900</td>
<td>4,630</td>
<td>930</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>DryBean</td>
<td>0</td>
<td>3,841</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Onions</td>
<td>1,600</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>Peas</td>
<td>1,300</td>
<td>0</td>
<td>0</td>
<td>600</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Potatoes</td>
<td>7,000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>SlowVeg</td>
<td>871</td>
<td>807</td>
<td>33</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>Spokra</td>
<td>0</td>
<td>1,200</td>
<td>360</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>Tompep</td>
<td>0</td>
<td>2,093</td>
<td>3,933</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>VegMix</td>
<td>1,694</td>
<td>1,788</td>
<td>1,658</td>
<td>2,799</td>
<td>3,637</td>
<td>2,002</td>
</tr>
<tr>
<td>12</td>
<td>Strawberry</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>750</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>Garlic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1,994</td>
</tr>
</tbody>
</table>

### Model Formulation and Notation

The four fundamental index sets of our model are as follows:

- \( \text{Crops} = \{1, 2, \ldots, 11\} \);
- \( \text{Fields} = \{1, 2, \ldots, 24\} \);
- \( \text{Months} = \{\text{Mar/Apr, May, June, July, Aug, Sept/Oct}\} \); and
- \( \text{Year} = \{1, 2, 3, 4\} \).
The set Crops does not contain Strawberries or Garlic (Crops 12 and 13), since these are perennial crops that we handle separately.

Our two main model parameters are

\[ ABF_j = \text{available bed feet in Field } j, \]  
\[ DEM_{ikl} = \text{demand of Crop } i \text{ in Year } k \text{ and Month } l. \]

In the following subsections, we detail the variables, parameters, objective function, and constraints of our model.

**Market Demand and Objective Function**

Our objective function and first set of constraints work together to attempt to meet the market demand as closely as possible. To accomplish this goal, we use

- a binary variable to keep track of when and where crops are planted,
- a continuous variable for the proportion of a field dedicated to a particular crop group, and
- a continuous variable for the amount of unmet market demand.

Specifically, we define the following variables:

\[ x_{ijkl} = \begin{cases} 1, & \text{if Crop } i \text{ is planted in Field } j \text{ in Year } k \text{ during Month } l; \\ 0, & \text{otherwise}. \end{cases} \]

\[ y_{ijkl} = \text{proportion of Field } j \text{ to Crop } i \text{ in Year } k \text{ during Month } l; \] and

\[ u_{ikl} = \text{unmet demand of Crop } i \text{ in Year } k \text{ during Month } l. \]

Recall that rather than maximize profit, our model focuses on meeting the projected market demand as closely as possible without violating any planting principles. We can accomplish this by minimizing the unmet demand with the following objective function:

\[ \text{minimize } Z = \sum_{i \in \text{Crops}} \sum_{k \in \text{Years}} \sum_{l \in \text{Months}} u_{ikl}. \]

The constraints associated with meeting the market demand are as follows, for every \( i \) in Crops, \( j \) in Fields, \( k \) in Years, and \( l \) in Months:

\[ y_{ijkl} \leq x_{ijkl}, \]  
\[ y_{ijkl} \geq 0.10 x_{ijkl}, \]  
\[ \sum_{j \text{ in Fields}} ABF_j y_{ijkl} \geq DEM_{ikl} - u_{ikl}, \]  
\[ u_{ikl} \geq 0, \]
where $x_{ijkl}$ is the “flag” variable that keeps track of when and where crops are planted.

Constraints (1) ensure that $y_{ijkl}$ can be nonzero only if $x_{ijkl}$ takes on the value 1, while constraints (2) ensure that a crop group can only be planted if it uses at least 10% of the field (it is undesirable to plant small amounts of a crop group in a field).

Constraints (3) and (4), coupled with the objective function, ensure that the market demand is met as closely as possible. That is, as the $u_{ikl}$ variables are pushed toward zero by the objective function, the rotation schedule gets closer to meeting demand (the demand for Crop $i$ in Year $k$ during Month $l$ is met if $u_{ikl} = 0$).

Constraints (1)–(4) do not ensure meeting the demand for strawberries or garlic—we handle them in a separate set of constraints to be described shortly.

**General Planting Principles**

Three important considerations that must be taken into account in developing a crop rotation plan are

- soil nutrient depletion,
- weed control, and
- the type of irrigation utilized.

We present these three considerations together because the modeling mechanisms used to achieve the goals are similar.

**Soil Nutrients**

Certain crop groups are “heavy nitrogen feeders” which tend to break down the soil and deplete its main nutrients. Therefore, it is advantageous to give the soil a rest between plantings of heavy feeding crops. Specifically, we ensure that crop groups designated as heavy feeders are not planted in the same field two years in a row.

**Weed Control**

Crop rotation is the main mechanism for weed suppression, since herbicides utilized in conventional farming often cannot be used on an organic farm. Specific crops can be classified as “weed contributing.” To disrupt the lifecycle of weeds, we establish the rule that if a crop designated as a weed contributor is planted in a particular field, then subsequent planting of weed contributors must be avoided for at least two years in that field.
Irrigation

Fields on the college farm can be individually tailored for either overhead or drip irrigation; but from a logistical standpoint, it is not practical to utilize both types of irrigation on the same field during a given year. Therefore, it is crucial that our crop rotation take the irrigation type of each crop group into account. While many crops can be watered using either drip or overhead irrigation, some crop groups can be irrigated only with drip irrigation, while others can be irrigated only with overhead irrigation. Thus, we must ensure that if a Drip-Only crop group is planted on a given field, then no Overhead-Only crop group can be planted on that field during that year. Similarly, if an Overhead-Only crop group is planted on a given field then no Drip-Only crop group can be planted on that field during that year.

In Table 3, we provide the classifications of our thirteen crop groups with respect to heavy feeders, weed contributors, and type of irrigation.

<table>
<thead>
<tr>
<th>Crop Group</th>
<th>Heavy-Feeder</th>
<th>Weed Contributor</th>
<th>Drip Irrigation Only</th>
<th>Overhead Irrigation Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brassica</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carrot</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cucurbits</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>DryBean</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Onions</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peas</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Potatoes</td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>SlowVeg</td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Spokra</td>
<td></td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Tompep</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>VegMix</td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Strawberry</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Garlic</td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

To enforce the planting principles and rules described above, we use binary flag variables to record whether a heavy feeder, weed contributor, or drip-only crop group is planted in a field during a given year:

\[
z_{jk} = \begin{cases} 
1, & \text{if a heavy-feeder crop is planted in Field } j \text{ during Year } k; \\
0, & \text{otherwise.}
\end{cases}
\]

\[
w_{jk} = \begin{cases} 
1, & \text{if a weed-contributor crop is in Field } j \text{ during Year } k; \\
0, & \text{otherwise.}
\end{cases}
\]

\[
r_{jk} = \begin{cases} 
1, & \text{if a crop requiring drip irrigation is in Field } j \text{ during Year } k; \\
0, & \text{otherwise.}
\end{cases}
\]
Using the information from Table 3, we define the following index sets:

- **HFC** = crops classified as heavy feeders = \{1, 3, 10, 12\};
- **WC** = crops classified as weed contributors = \{3, 5, 9, 10, 12, 13\};
- **DC** = crops classified as drip-only irrigation = \{2, 4, 7, 11, 12, 13\}; and
- **OC** = crops classified as overhead irrigation only = \{3, 8, 9, 10\}.

The constraints needed to ensure our feeding, weed control, and irrigation type rules are as follows, where \(M\) is a large value:

\[
\sum_{h \in \text{HFC}} \sum_{l \in \text{Months}} x_{hjl} \leq Mz_{jk}, \quad \forall \ j \text{ in Fields, } k \text{ in Years}; \quad (5)
\]

\[
\sum_{h \in \text{HFC}} \sum_{l \in \text{Months}} x_{h,j,k+1,l} \leq M(1 - z_{jk}), \quad \forall \ j \text{ in Fields, } k \text{ in Years} - \{4\}; \quad (6)
\]

\[
\sum_{h \in \text{WC}} \sum_{l \in \text{Months}} x_{hjl} \leq Mw_{jk}, \quad \forall \ j \text{ in Fields, } k \text{ in Years}; \quad (7)
\]

\[
\sum_{h \in \text{WC}} \sum_{l \in \text{Months}} x_{h,j,k+1,l} \leq M(1 - w_{jk}), \quad \forall \ j \text{ in Fields, } k \text{ in Years} - \{4\}; \quad (8)
\]

\[
\sum_{h \in \text{WC}} \sum_{l \in \text{Months}} x_{h,j,k+2,l} \leq M(1 - w_{jk}), \quad \forall \ j \text{ in Fields, } k \text{ in Years} - \{3, 4\}; \quad (9)
\]

\[
\sum_{h \in \text{DC}} \sum_{l \in \text{Months}} x_{hjl} \leq Mr_{jk}, \quad \forall \ j \text{ in Fields, } k \text{ in Years}; \quad (10)
\]

\[
\sum_{d \in \text{OC}} \sum_{l \in \text{Months}} x_{d,j,k+1,l} \leq M(1 - r_{jk}), \quad \forall \ j \text{ in Fields, } k \text{ in Years}. \quad (11)
\]

Constraints (5) and (6) separate heavy-feeding crops from subsequent planting for at least one year: If a heavy feeder is planted in Field \(j\) during Year \(k\), then no heavy feeder can be planted in Field \(j\) during Year \(k + 1\).

Constraints (7)–(9) ensure that if a weed-contributing crop is planted in Field \(j\) during Year \(k\), then weed-contributing crops cannot be planted in Field \(j\) during Years \(k + 1\) and \(k + 2\).

Constraints (10) ensure that if there is a Drip-Only crop group planted on Field \(j\) during Year \(k\), then all the crop groups planted on that field during Year \(k\) must be amenable to drip-irrigation.

Finally, constraints (11) ensure that if an Overhead-Only crop group is planted on Field \(j\) during Year \(k\), then all crops planted on that field must be amenable to overhead irrigation. Note that while we need two separate sets with respect to drip- and overhead-only irrigation, we only need one binary flag variable to keep track of both (we arbitrarily chose to keep track of overhead irrigation).
Perennial Crops: Garlic and Strawberries

Garlic and strawberries are perennial crops, which means that they stay in the ground for multiple planting seasons. These crops present a challenge since our model is built based on annual crops. Garlic is an overwinter crop, so our crop rotation must keep track of the garlic that remains planted in the soil from the previous year. Strawberries remain in the soil for a year and a half, so our crop rotation must keep track of the strawberries that remain from the two previous years.

Since these crops behave differently from our other crop groups, we use separate variables to keep track of them, as opposed to grouping them with the \( x_{ijkl} \) variables. Specifically, we define \( s_{jk} \) and \( g_{jk} \) as follows:

\[
\begin{align*}
    s_{jk} &= \begin{cases} 
        1, & \text{if 750 bed feet of strawberries are in Field } j \text{ during Year } k; \\
        0, & \text{otherwise}; 
    \end{cases} \\
    g_{jk} &= \begin{cases} 
        1, & \text{if 1,994 bed feet of garlic are in Field } j \text{ during Year } k; \\
        0, & \text{otherwise}. 
    \end{cases}
\end{align*}
\]

The constraints that ensure the behavior of these crops are as follows:

\[
\sum_{j \text{ in Fields}} s_{jk} = 1, \quad \forall \ k \text{ in Years}; \tag{12}
\]

\[
s_{jk} + s_{j,k+1} + s_{j,k+2} \leq 1, \quad \forall \ j \text{ in Fields}, \ k \text{ in Years}; \tag{13}
\]

\[
s_{jk} + s_{j,k-1} + s_{j,k-2} \leq r_{jk}, \quad \forall \ j \text{ in Fields}, \ k \text{ in Years}; \tag{14}
\]

\[
\sum_{j \text{ in Fields}} g_{jk} = 1, \quad \forall \ k \text{ in Years}; \tag{15}
\]

\[
g_{jk} + g_{j,k+1} \leq 1, \quad \forall \ j \text{ in Fields}, \ k \text{ in Years}; \tag{16}
\]

\[
g_{jk} + g_{j,k-1} \leq r_{jk}, \quad \forall \ j \text{ in Fields}, \ k \text{ in Years}. \tag{17}
\]

To meet market demand, we must plant 750 bed feet of strawberries and 1,994 bed feet of garlic each year, which are enforced by constraints (12) and (15).

Constraints (13) ensure that if strawberries are planted in Field \( j \) during Year \( k \), then strawberries cannot be planted again in Field \( j \) for the next two years; while constraints (16) ensure that if garlic is planted in Field \( j \) during Year \( k \) then garlic cannot be planted again in Field \( j \) the following year.

Constraints (14) and (17) ensure both strawberries and garlic are planted in a field that will use drip irrigation during the time that they are in the ground.
Planting Principles for Pest Control

Insect management can be challenging for organic farmers because they cannot use industrial pesticides. Crop rotation can be effective at controlling pests because changing the location of crops can prevent the build-up of pests within the soil by disrupting their life cycles. Controlling pests helps to improve crop yield and the quality of the produce, while also reducing labor requirements.

Our rotation model prevents the same crop groups from being planted in the same field two years in a row except for VegMix, which is not susceptible to pests. In addition, the farm managers have determined that some families of crop groups are more susceptible to pests than others and therefore we must eliminate subsequent same-crop plantings for more than a year. In particular,

- subsequent plantings of brassica in the same field must be at least 2 years apart,
- subsequent plantings of cucurbits in the same field must be at least 3 years apart,
- subsequent plantings of tompep and potatoes in the same field must be at least 3 years apart, and
- plantings of onions and garlic in the same field must be at least 3 years apart.

To enforce these rules, we use the following flag variables:

\[
f_{ijk} = \begin{cases} 
1, & \text{if Crop } i \text{ is planted in Field } j \text{ during Year } k; \\
0, & \text{otherwise}, 
\end{cases}
\]

\[
b_{jk} = \begin{cases} 
1, & \text{if brassica is planted in Field } j \text{ during Year } k; \\
0, & \text{otherwise}, 
\end{cases}
\]

\[
c_{jk} = \begin{cases} 
1, & \text{if cucurbits are planted in Field } j \text{ during Year } k; \\
0, & \text{otherwise}, 
\end{cases}
\]

\[
t_{jk} = \begin{cases} 
1, & \text{if tompeps or potatoes are planted in Field } j \text{ during Year } k; \\
0, & \text{otherwise}, 
\end{cases}
\]

\[
o_{jk} = \begin{cases} 
1, & \text{if onions or garlic are planted in Field } j \text{ during Year } k; \\
0, & \text{otherwise}. 
\end{cases}
\]

The constraints that ensure these rules are as follows, for all \(i\) in Crops, \(j\) in Fields, and \(k\) in Years:
Constraints (18) and (19) prevent the same crop group from being planted in the same field two years in a row except for VegMix (Crop 11).

Constraints (20)–(22) ensure that subsequent plantings of brassica are two years apart, while constraints (23)–(24) ensure that subsequent plantings of cucurbits are three years apart.

Constraints (25) and (26) ensure that subsequent plantings of tomatoes and potatoes are three years apart, and finally constraints (27)–(30) ensure that subsequent plantings of onions and garlic are three years apart.

### Planting Principles: Fallow Periods, Minimum Plantings, Field Capacity

Leaving a field uncropped, or bare fallow, is an important mechanism in crop rotation that rebuilds soil fertility and helps ensure land productivity. We have chosen to require that every field is left unplanted at least once during the four-year period. To accomplish this, we use the following flag variable:

\[ p_{jk} = \begin{cases} 
1, & \text{if a crop is planted in Field } j \text{ during Year } k; \\
0, & \text{otherwise.}
\end{cases} \]
The following constraints enforce the definition of $p_{jk}$ and maintain the spatial capacity of the fields, for all $j$ in Fields and $k$ in Years:

$$
\sum_{i \text{ in Crops}} \sum_{l \text{ in Months}} y_{ijkl} \leq p_{jk} - \frac{750}{ABF_j} (s_{j,k-2} + s_{j,k-1} + s_{j,k}) - \frac{1994}{ABF_j} (g_{j,k-1} + g_{j,k})
$$

(31)

For any crops to be planted in Field $j$ during Year $k$, $p_{jk}$ must equal 1. Thus, by constraints (31), the total proportion of Field $j$ that is cropped during Year $k$ must be less than or equal to 1 minus the proportion of the field taken up by strawberries and garlic.

Now, in order to ensure that every field is left unplanted at least once during the four-year period, we introduce the following constraints:

$$
p_{j1} + p_{j2} + p_{j3} + p_{j4} \leq 3, \quad \forall \ j \text{ in Fields.}
$$

Planting Principles: No Planting Gaps

For logistical reasons, it is desirable to keep fields active only during consecutive months. For example, we would not want to plant crops in Field 1 in May and then again in July without planting during June. Moreover, it is important that no field should be active for more than 3 months during the year. To enforce these rules we make use of the following flag variable:

$$
n_{jkl} = \begin{cases} 
1, & \text{if a crop is planted in Field } j \text{ at Year } k \text{ during Month } l; \\
0, & \text{otherwise.}
\end{cases}
$$

Then the following constraints eliminate the planting gaps, where the sums are over all $j$ in Fields, $k$ in Years, and $l$ in Months:

$$
\sum_{i \text{ in Crops}} x_{ijkl} \leq M n_{jkl}, \quad \forall \ j \text{ in Fields, } k \text{ in Years, and } l \text{ in Months.}
$$

(32)

$$
\sum_{i \text{ in Crops}} x_{ijkl} \geq n_{jkl},
$$

(33)

$$
n_{jk1} + n_{jk3} - n_{jk2} \leq 1, \quad \forall \ j \text{ in Fields, } k \text{ in Years, and } l \text{ in Months.}
$$

(34)

$$
n_{jk1} + n_{jk4} - n_{jk2} - n_{jk3} \leq 1, \quad \forall \ j \text{ in Fields, } k \text{ in Years, and } l \text{ in Months.}
$$

(35)

$$
n_{jk1} + n_{jk5} - n_{jk2} - n_{jk3} - n_{jk4} \leq 1, \quad \forall \ j \text{ in Fields, } k \text{ in Years, and } l \text{ in Months.}
$$

(36)

$$
n_{jk1} + n_{jk6} - n_{jk2} - n_{jk3} - n_{jk4} - n_{jk5} \leq 1, \quad \forall \ j \text{ in Fields, } k \text{ in Years, and } l \text{ in Months.}
$$

(37)

$$
n_{jk2} + n_{jk4} - n_{jk3} \leq 1, \quad \forall \ j \text{ in Fields, } k \text{ in Years, and } l \text{ in Months.}
$$

(38)

$$
n_{jk2} + n_{jk5} - n_{jk3} - n_{jk4} \leq 1, \quad \forall \ j \text{ in Fields, } k \text{ in Years, and } l \text{ in Months.}
$$

(39)
Note that constraints (32) and (33) ensure the definition of $n_{jkl}$, while constraints (44) ensure that fields are active for only a maximum of three months a year.

Constraints (34)–(43) enforce that a field must be active in consecutive months. For example, constraints (34) ensure that a field cannot be active during Month 1 (Mar/Apr) and Month 3 (June) without being active during Month 2 (May), while constraints (35) enforce that a field cannot be active during Month 1 (Mar/Apr) and Month 4 (July) without being active during Months 2 and 3.

**Planting Principles: Planting Windows**

In the organic farming system, crops are split into two broad categories: cash crops and cover crops.

- Cash crops are usually grown for profit and provide the majority of the revenue for the farm.
- Cover crops are planted primarily to manage soil fertility, soil quality, water, weeds, pests, diseases, and biodiversity. In most cases, cover crops do not provide any monetary revenue.

The college farm plants most of its cover crops during the winter or early spring. However, it is imperative that fields be left unplanted for a couple of months, whether this is at the end of the planting year or earlier the next following year.

Our model takes into account this planting delay by categorizing fields as being “early” or “late” planted fields. Early planted fields contain crops planted in Month 1 (Mar/Apr) or 2 (May). Late planted fields contain crops planted in Months 4 (July), 5 (Aug), or 6 (Sept/Oct).

To ensure a planting window for cover crops, our rotation model eliminates the possibility that a field that is planted Late is then planted Early during the following planting season. The flag variables used to keep track of this information are:

$$
early_{jk} = \begin{cases} 
1, & \text{if a crop is planted “early” in Field } j \text{ during Year } k; \\
0, & \text{otherwise.}
\end{cases}
$$
late\textsubscript{jk} = \begin{cases} 
1, & \text{if a crop is planted “late” in Field } j \text{ during Year } k; \\
0, & \text{otherwise.}
\end{cases}

Keeping in mind that Months 1 and 2 are considered “early,” while Months 4, 5, and 6 are considered “late,” the following constraints enforce the early and late flag definitions:

\begin{align*}
   n_{jk1} + n_{jk2} & \leq M \cdot early\textsubscript{jk}, & \forall \ j \text{ in Fields, } k \text{ in Years}, \\
   n_{jk4} + n_{jk5} + n_{jk6} & \leq M \cdot late\textsubscript{jk} & \forall \ j \text{ in Fields, } k \text{ in Years}.
\end{align*}

The following constraints ensure that crops planted during early months cannot follow crops planted during late months from the previous year:

\[ early\textsubscript{j,k+1} + late\textsubscript{jk} \leq 1, \quad \forall \ j \text{ in Fields, } k \text{ in Years}. \]

### Other Considerations

Other considerations were taken into account in constructing our model. However, for the sake of brevity, we mention these considerations only briefly without providing the full details of how they were implemented in our model.

- While our model does not allow gaps between planting periods during the same year, we do allow “double cropping,” which is the practice of growing crops on the same area of a field at different times during the same year. Specifically, we allow certain crops to be planted in Month 1 (Mar/Apr) and then other crops to be planted in the same section of the field in Month 6 (Sept/Oct) during the same year. This complication was handled through the use a binary flag variable and a modification of many of our constraints.

- Our model prohibits crops from being split over multiple fields during the same planting season, with a few exceptions (such as potatoes, because of the large market demand for this crop).

- Our model attempts to utilize fully any active field before planting in another field. For example, the model tries to prevent two fields from being partially utilized—one field should be close to fully utilized before any other fields are used.

- Certain crops are not considered compatible and therefore our model restricts these crop groups from being planted in the same field during the same year.

- Finally, our model needs to take into consideration past planting decisions.
Results and Conclusions

Our final model, with more than 9,000 variables and 13,000 constraints, was formulated using the Mosel modeling language and solved using the Xpress 7.0 optimizer on a Dell precision workstation. We had to allow some small violations of the planting principles during the first year to obtain a feasible solution, since we had to take into account the planting decisions made prior to our study. Unfortunately, since our model is rigid in terms of planting principles, we had to make these adjustments manually to the violated constraints in the model to ensure feasibility.

The solver was able to find the optimal solution to our model within a few minutes, and the resulting crop rotation fully met market demand. That is, the optimal solution had the desired objective value of 0, with $u_{ikl} = 0$ for all $i$, $k$, and $l$. In Table 4, we provide the output from our model that specifies the field summary information during the four-year rotation schedule.

Table 4.

<table>
<thead>
<tr>
<th>Field Information over the 4 year rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year: 1</td>
</tr>
<tr>
<td>Resting Fields: 6, 7, 12</td>
</tr>
<tr>
<td>Heavy Feeder Fields: 2, 4, 8, 11, 15, 17, 18, 19, 20</td>
</tr>
<tr>
<td>Weed Crop Fields: 2, 4, 11, 15, 17, 19, 20</td>
</tr>
<tr>
<td>Year: 2</td>
</tr>
<tr>
<td>Resting Fields: 5, 7, 8, 10, 17, 18, 19, 24</td>
</tr>
<tr>
<td>Heavy Feeder Fields: 1, 3, 9, 12, 23</td>
</tr>
<tr>
<td>Weed Crop Fields: 1, 9, 16, 23</td>
</tr>
<tr>
<td>Year: 3</td>
</tr>
<tr>
<td>Resting Fields: 1, 2, 3, 11, 13, 15, 18</td>
</tr>
<tr>
<td>Heavy Feeder Fields: 5, 6, 8, 10, 20, 22</td>
</tr>
<tr>
<td>Weed Crop Fields: 5, 8, 10, 12, 22</td>
</tr>
<tr>
<td>Year: 4</td>
</tr>
<tr>
<td>Resting Fields: 4, 6, 9, 14, 16, 20, 21, 22, 23</td>
</tr>
<tr>
<td>Heavy Feeder Fields: 2, 3, 7, 11, 17, 18, 19</td>
</tr>
<tr>
<td>Weed Crop Fields: 2, 3, 7, 11, 15, 17, 18, 19</td>
</tr>
</tbody>
</table>

A close examination of the output reveals that the planting principles are maintained:

- Every field was left fallow at least once during the four-year rotation (with Fields 6, 7, and 18 resting twice).
- Any field in which a weed-contributing crop was planted has at least a two-year gap before another weed contributing crop is planted in that same field. For example, during Year 1 we see that Field 2 has weed
contributing crops and that it is not until Year 4 before another weed contributing crop is planted in that same field.

- Similarly, fields in which a heavy feeding crop is planted have at least a one-year gap before another heavy feeding crop is planted.

In the Appendix, we provide the actual planting schedule for the first year.

The college farm used our rotation plan in 2013, with a few minor modifications to address some unexpected soil conditions that occurred during the beginning of the planting season. The farm managers were pleased that the rotation was able not only to meet market demand, but also was able to ensure that none of the planting principles were violated over the four-year schedule. The rotation scheduling used previously typically produced rotations that violated these principles, because of the difficulty of foreseeing the consequences of planting choices. These violations often resulted in crop loss due to disease and pests, along with additional labor requirements to address weed issues.

It is difficult to quantify the reduction in labor and the improvement in produce quality achieved by using the crop rotation plan developed by our model. However, the farm managers were confident that our rotation schedule was superior to the one previously in use, because our rotation fully conformed to the planting principles that they had outlined, which had not previously been made explicit.

While the college farm is not currently using our rotation schedule, the farm managers found the process of developing the planting principles to be invaluable. This process helped to turn intuition and experience into tangible principles that could be applied to current and future crop rotation practices.

The Future

Shortly after our initial study, the college farm acquired an additional 14 fields, which greatly increased available bed feet. While additional fields and updated market demand information could easily be updated in our model, the farm is interested in making additional changes in crop rotation. Specifically, the farm managers want to group fields into “blocks” and make planting decisions based on these blocks. Therefore, planting decisions would now require rules that take both the individual fields and block information into account. Moreover, the managers are interested in simplifying how crops are rotated so as to simplify overall management of the farm.

Taking all these new considerations into account would mean significantly changing the model. Since this work was a research project that involved a graduating student, these changes have yet to be implemented.
This situation, of course, illustrates an important consideration that must be taken into account with student-faculty research projects: Who will keep the model current once the student has graduated? In addition to implementing these new ideas, there are a number of other changes that we hope could be incorporated into the model:

- The fact that our model was able to fully satisfy the market demand implies that we could enforce meeting the demand using \textit{hard} constraints (which cannot be violated), as opposed to using the objective function and \textit{soft} constraints (which can be violated) as we do in our current model. This in turn would allow us to utilize the objective function in a more effective manner. For example, we could use the objective function to maximize the additional profit that could be realized by planting “cash crops” in unused field space.

- We could extend the planning horizon beyond four years, so as to improve the model’s handling of end-effects and better coordinate the types of irrigation used on adjacent fields.

The farm managers have recently expressed interest in revisiting this project, and we are hopeful that another student will take up the mantle to update the crop rotation model.

\section*{References}


Dury, Jérôme, Noémie Schaller, Frédéric Garcia, Arnaud Reynaud, and Jacques Eric Bergez. 2012. Models to support cropping plan and crop rotation decisions. A review. \textit{Agronomy for Sustainable Development} 32: 567–580. \url{https://hal.archives-ouvertes.fr/hal-00930508/file/hal-00930508.pdf}.


## Appendix

Below we provide the output from our model that specifies the quantity and location of each crop group during the first year.

**Production Output by Field**

**Year 1**

**Mar/Apr**

- **Brassica** (365 total bf): 365 in F4
- **Carrot** (433 total bf): 433 in F9
- **Onion** (1600 total bf): 1600 in F16
- **Peas** (1300 total bf): 1300 in F21
- **Potato** (7000 total bf): 1346 in F1, 2506 in F10, 685 in F16, 2463 in F24
- **SlowVeg** (871 total bf): 483 in F4
- **VegMix** (1694 total bf): 1694 in F9
- **Strawberries** (750 total bf): 750 in F11

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**May**

- **Carrot** (767 total bf): 767 in F13
- **Cucurbit** (900 total bf): 900 in F4
- **Dry Bean** (3841 total bf): 3841 in F1
- **SlowVeg** (807 total bf): 765.81 in F4, 41.19 in F11
- **Spokra** (1200 total bf): 1200 in F4
- **Tompep** (2093 total bf): 1330.5 in F2, 762.5 in F20
- **VegMix** (1788 total bf): 788.23 in F9, 999.77 in F13

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**June**

- **Brassica** (200 total bf): 200 in F18
- **Carrot** (1500 total bf): 1448.5 in F3, 51.5 in F13
- **Cucurbit** (4630 total bf): 2968 in F15, 1662 in F19
- **SlowVeg** (33 total bf): 33 in F11
- **Spokra** (360 total bf): 360 in F19
- **Tompep** (3933 total bf): 1848 in F2, 2085 in F20
- **VegMix** (1658 total bf): 262.84 in F3, 1395.16 in F14

---
July

Brassica (1475 total bf): 164.74 in F11, 1310.26 in F18
Carrot (1900 total bf): 1591.51 in F3, 308.49 in F18
Cucurbit (930 total bf): 930 in F19
Peas (600 total bf): 600 in F21
SlowVeg (100 total bf): 100 in F19
VegMix (2799 total bf): 889.19 in F14, 1909.81 in F23
-------

Aug

Brassica (1975 total bf): 81 in F8, 1894 in F17
Carrot (533 total bf): 533 in F5
SlowVeg (100 total bf): 100 in F17
VegMix (3637 total bf): 2683.37 in F5, 953.63 in F23
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Sep/Oct

VegMix (2002 total bf): 2002 in F8
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