Using Schlieren Optics To Visualize Ultrasonic Standing Waves In Acoustic Levitation

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Using Schlieren Optics To Visualize Ultrasonic Standing Waves In Acoustic Levitation

Submitted in partial fulfillment of honors requirements

for the Department of Physics and Astronomy, Dickinson College,

by

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May 18, 2020
Great are the works of the LORD, studied by all who delight in them.

Psalm 111:2
Abstract

Acoustic levitation is the phenomenon wherein a standing sound pressure wave can be used to levitate small objects by exerting a force large enough to counter the force of gravity. In this paper we examine the theory of acoustic levitation using an analysis from first-principles, a ponderomotive analysis, and a fluid motion analysis. The first-principles approach involves using Newton’s second law to calculate the net force of an acoustic standing pressure wave on a small object. However, the rapid oscillation of the pressure wave results in a time averaged net force of zero, so we extend the analysis by separating out the rapid motion of the particle from the slow motion. Doing so leads to a ponderomotive force that has a promising form, since the force no longer averages to zero, but disagrees with the research literature. Hence, we consider the accepted analysis using the governing equations of fluid motion and find that the acoustic radiation force causes objects to rest at the pressure nodes of a standing sound pressure wave.

The remainder of this paper describes a three-phase investigation into acoustic levitation and schlieren optics. Phase one consists of building a schlieren optics experiment. The schlieren apparatus consists of a point source of light, which is focused by a concave spherical mirror onto a light-block in front of a camera. Changes in refractive index in the path of the light rays will deflect the light rays around the light-block into the camera. This allows for the visualization of minute density gradients in air, allowing us to observe, for example, the airflow above a lit candle or the flow of air from a hair dryer. Phase two of the project involves experimentally producing acoustic levitation. To do this, we use an ultrasonic transducer driven with a 30.1 kHz sinusoidal signal and directed at a flat piece of glass. The resulting standing wave is capable of supporting several small Styrofoam balls. Finally, in phase three we determine the location of these levitating objects using three different analyses: a geometric comparison, direct measurements of the standing pressure wave, and a visual approach using the schlieren system. These three analyses are consistent and demonstrate that objects levitate just below the nodes of standing pressure waves.
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1 Introduction

Light and sound, being two of the primary means of interacting with the physical world, have been studied for centuries if not millennia. Though we have not always understood the nature of light and sound, and perhaps still do not, we have investigated their properties because they have been central to daily function and even survival. The earliest investigations of sound were conducted by the Greeks and expanded upon by the Romans and Arabs [1]. One example is Pythagoras, who is supposed to have investigated why some sounds are more pleasant when combined, and who formulated a theory of harmonics using standing waves on strings of different lengths [2]. Similarly, light also has a long history of scientific study beginning with the ancient Greeks and subsequently the Arabs [3]. While some early beliefs, such as Plato’s theory that light is emitted from the eye and allows the viewer to perceive [4], are now known to be false, other early scientists such as Abu Sad Ibn-Sahl in the 10th century studied the focusing properties of lenses and his work demonstrates an understanding of what would later become Snell’s law of refraction [5].

1.1 Refraction of Light

The phenomenon of refraction is the deflection of light rays from a straight path as they enter a new medium. As light enters an optically denser medium, it is deflected towards the normal of the interface between the media. Conversely, as light enters an optically less dense medium, it is deflected away from the normal. Within the realm of optics, refraction has been particularly widely studied, partly because of the ubiquity of the phenomenon and partly because it is quite fascinating. Water and fire are two essential resources that were necessary for societies to grow, and both provided ample opportunity to observe this phenomenon. As is commonly observed, images of objects in water appear shifted and distorted with respect to the surroundings. Similarly, the area above a fire appears to shimmer as the light traveling through this region is distorted. In addition to Ibn-Sahl, Rene Descartes and Willebrord Snellius also independently derived the laws of refraction [6], and while Snellius did not publish his result, the law of refraction is credited to him because he derived it earlier than Descartes. The simple formulation of the law of refraction is given by

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 , \]  

where \( n_1 \) is the refractive index of the first medium, \( n_2 \) is the refractive index of the second medium, \( \theta_1 \) is the angle of incidence, and \( \theta_2 \) is the angle of the refracted ray, both measured with respect to the normal. This result can be derived from Fermat’s principle of least time, which states that light takes the path of least time to travel between two points [7]. Interestingly, the path of least time may not be the path of shortest distance when light travels from one medium to another—
a result caused by the change in speed in the two materials. Hence, the path of least time need not necessarily be a straight line when there is an intermediate change in medium of propagation, which leads to the “bending” of light between the two materials of propagation.

Until the 19th century, refraction was studied by using different media to examine the nature and properties of light. In the mid-19th century, with the development of the schlieren imaging system by August Toepler, this process was reversed and the phenomenon of refraction began to be used to study the nature and properties of physical objects and phenomena [8]. Put simply, schlieren imaging allows for the visualization of minute changes in the refractive indices of media. One example is Hubert Schardin’s (1902–1965) famous image of the shock waves caused by a bullet [9], shown in Fig. 1. This photograph involved imaging techniques capable of reaching a million frames per second, which was an impressive feat for this time period. The name “schlieren” originates from the German word for “streaks,” which is indicative of the type of images formed as they appear to be streaked with gradients of light.

Presently, the refraction of light is a standard topic covered in most high school and introductory college physics curricula. Common demonstrations include mapping the path of a light ray as it passes through rectangular or circular transparent objects, or using a drop of water as a magnifying glass. The theoretical portion to such introductory courses might discuss ray diagrams of lenses and mirrors using the lens-maker’s formula. Schlieren imaging is sufficiently complex that it is not typically discussed at the high-school or undergraduate level. However, it is relatively easy to understand from a qualitative perspective and can provide striking images that allow one to “see the invisible.”
1.2 Schlieren Imaging

We shall consider the schlieren apparatus shortly, but let us first examine the theory behind the technique. Light slows down as it interacts with matter and propagates at a constant speed through homogeneous media. Thus, it propagates fastest in a vacuum and slower in different media. The index of refraction $n$ is defined by the ratio of the speed of light propagating through the material $v$ to the speed of light in a vacuum $c$:

$$n = \frac{c}{v}.$$  \hspace{1cm} (2)

For gases, which are the most common media used in schlieren imaging, there is a linear relationship between mass density $\rho$ and the refractive index $n$ given by

$$n - 1 = k\rho,$$  \hspace{1cm} (3)

where $k$ is the Gladstone-Dale coefficient ($k \approx 0.23 \text{ cm}^3/\text{g}$ for air under standard conditions) [10, 11]. It is important to note that $n$ and $\rho$ are only weakly related; at Standard Temperature and Pressure (STP), if the density of air changes by two orders of magnitude the refractive index changes by less than 3%. Thus, detecting density variations in air requires a very sensitive apparatus.

As light enters a new medium, part of it is reflected, part of it is absorbed, and the remainder continues to propagate through the new medium. The light that propagates not only changes speed but also changes direction based on the angle of incidence of the light ray according to Snell’s law. To understand why the light ray bends, consider how the wave-fronts of the wave slow down in a non-uniform manner according to which medium each part of the wave-front is in. For example, consider a ray of light travelling from a vacuum into an optically denser medium (see Fig. 2).

![Figure 2: Left: A ray diagram of light in a vacuum entering an optically denser medium. Right: The ray with its wave-fronts (not to scale).](image)
As the lower part of the wave-front enters the denser medium first, it slows down before the rest of the wave-front, causing the ray to “bend” towards the normal. Since the frequency is the same in the two materials, the wavelength must change according to $v = f\lambda$. The result is that the wave-fronts are closer to each other in the denser medium. Similarly, a ray moving from a more dense medium to a less dense medium will bend away from the normal as the speed changes non-uniformly along the wave-front. A ray that is incident perpendicular to a surface will not experience any redirection because the wave-front is uniformly incident on the surface of the new medium.

Schlieren imaging is a technique that allows the visualization of deflected light due to density changes in the medium through which it travels. A schlieren apparatus, shown in Fig. 3, consists of a concave spherical mirror with a long focal-length $f$ that is used to focus a point source of light that is placed at a distance $2f$ from the mirror. For an object near the principal axis of a spherical mirror, the object distance $d_o$ and the image distance $d_i$ are related according to

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}. \tag{4}$$

Thus, for the present situation the image will be formed quite close to the light source, a distance $2f$ from the mirror and reflected about the principal axis (see Fig. 3). A thin wire is stretched across the image point and a video camera is placed behind the wire. If the medium in front of the mirror is undisturbed, no light from the point source should enter the camera lens. However, if the density of the medium through which the light travels changes, the light will be refracted and will “bend” around the light-block and enter the camera. In this way, even minute density changes, such as air currents from a burning candle, can be visualized.

Figure 3: Schematic diagram of our schlieren apparatus (not to scale), taken from Fig. 1 of Ref. [12]
1.3 Standing waves

One method of creating density gradients that may be studied using a schlieren system is by heating (or cooling) the air. For example, a burning candle will create hot air that rises due to a decrease in density. The gas temperature can be related to density using the ideal gas law $PV = nRT$ where $P$ is the pressure, $V$ is the volume, $n$ is the number of moles, $R$ is the universal gas constant, and $T$ is the absolute temperature. Using the fact that $n = m/M$, where $m$ is the mass and $M$ is the molecular mass, the density can be found to be

$$\rho = \frac{m}{V} = \frac{MP}{RT}.$$  \hfill (4)

This equation indicates that for a constant pressure, temperature is inversely proportional to density; as temperature increases the density decreases and vice versa.

Another way of introducing density gradients is by using sound waves. Sound waves are longitudinal waves comprised of alternating zones of compressions and rarefactions, corresponding to high-density and low-density regions. Using Eq. (4) we see that for a fixed temperature, density is proportional to pressure. However, imaging sound is complicated by the fact that sound travels at approximately 343 m/s in air. Visualizing a travelling sound wave would therefore require a high-speed camera, which can be very expensive. Hence, it is simpler to image a standing wave because its “location” remains fixed.

A standing sound wave can be set up by driving a speaker with a sinusoidal oscillation and directing it at a flat reflector. With the appropriate separation, the reflected wave and the original wave interfere to form a standing wave. Mathematically, a standing wave results from the superposition of two waves of the same amplitude and frequency travelling in opposite directions. For example, consider the superposition of two pressure waves traveling in opposite directions

$$p_{st}(x,t) = p_t[\cos(kx + \omega t) + \cos(kx - \omega t)],$$  \hfill (5)

where $p_t$ is the amplitude of the traveling waves. Expanding the trigonometric functions and simplifying leads to

$$p_{st}(x,t) = 2p_t \cos(kx) \cos(\omega t).$$  \hfill (6)

In this form, we can see that the superposed wave is effectively a cosine curve whose “amplitude” varies between $+2p_t$ and $-2p_t$. Thus, the amplitude of a standing wave is twice as large as the amplitude of the original traveling waves.

As a standing wave oscillates in time, crests and troughs oscillate back and forth each half period. The rapid oscillation of the standing wave creates the appearance of a series of anti-nodes (crests/troughs) separated by nodes, which creates the appearance of a shorter wavelength.
Viewing the full waveform would therefore require a high-speed camera, which, as mentioned earlier, is impractical. However, we can overcome this obstacle by using a light source that is strobed at the same frequency as the soundwave to image the entire standing wave (including both troughs and crests) at a single phase.

1.4 Overview of Project

This project can be divided into three separate components. The first component consists of using the Schlieren system to image density gradients in air. This portion involves setting up the schlieren apparatus, tuning it to be able to visualize density changes, and using colored filters to distinguish between positive and negative density gradients. The second component involves ultrasonic standing waves. Specifically, we generate a wave with a sufficiently large amplitude to levitate small objects. This phenomenon is known as acoustic levitation and will be discussed in the next section. Finally, the third component is to use schlieren imaging to investigate the locations at which the particles are levitated in the standing wave.

2 Acoustic Levitation

Acoustic levitation is the phenomenon whereby an object is levitated using sound. Typically, this is achieved using an ultrasonic standing wave. When the force exerted by the standing wave on the objects is large enough to counteract the gravitational force, the object comes to rest at an equilibrium point. Figure 4 shows a picture of Styrofoam balls being levitated in an acoustic standing wave.

![Figure 4: Acoustic levitation of Styrofoam balls in an ultrasonic standing wave. In this experiment the frequency of the sound wave is 30.1 kHz, resulting in a wavelength of 1.14 cm. As will be explained later, the spacing of the balls is equal to half this wavelength.](image)
Acoustic levitation is not a recently discovered phenomenon, having been proposed in 1933 by Bück and Müller [13]. Until recently this phenomenon was used mostly in combination with spectroscopy or in process engineering [14]. However, recent technological advances have made it relatively easy to obtain acoustic levitation, so the phenomenon has gained interest. The use of ultrasonic frequencies ensures the wavelength is small enough to visualize the phenomenon conveniently and exempts one from having to hear the rather unpleasant loud noises of audible frequencies.

Understanding acoustic levitation turns out to be surprisingly subtle. We begin by considering a simple, first-principles approach for analyzing acoustic levitation. As we shall see, the reality is more complex than a first-principles approach would suggest, and the reason for this is not immediately apparent.

2.1 First-Principles Analysis

In an introductory physics class, students use Newton’s second law of motion to analyze the behavior of objects subjected to various forces. In the present situation, let us consider an object of mass \( m \) in the shape of a cube of edge length \( 2a \) located in a standing pressure wave at an arbitrary location \( x \) (see Fig. 5). Since we are mainly interested in the effect of the sound wave, we will neglect gravity in our analysis.

As described earlier, a standing pressure wave is described as in Eq. (6). Including the ambient pressure \( p_{\text{amb}} \), the cube in Fig. 5 lies within a pressure field given by

\[
p(x, t) = p_{\text{amb}} + p_0 \cos(kx) \cos(\omega t),
\]

where \( p_0 = 2p_t \) is the amplitude of the standing pressure wave. Because of the symmetry of the cube and the pressure field, the forces exerted by the wave on the cube in the \( y \) and \( z \) directions

\[
\text{Figure 5: A cube of mass } m \text{ and edge length } 2a \text{ is located with its center at a position } x \text{ in a standing pressure wave, represented by the solid/dashed curves.}
\]
will cancel out. Neglecting air resistance, the net force on the object is then simply the sum of the pressure times the area on the left and right:

\[ F_{\text{net}} = 4a^2\{p_0 \cos[k(x - a)] \cos(\omega t) - p_0 \cos[k(x + a)] \cos(\omega t)\}. \] (8)

Expanding the cosine functions and simplifying the expression leads to

\[ F_{\text{net}} = 8p_0a^2 \sin(kx) \sin(ka) \cos(\omega t). \] (9)

Substituting into Newton’s second law, the equation of motion is then

\[ m\ddot{x} = 8p_0a^2 \sin(kx) \sin(ka) \cos(\omega t) \] (10)

Note that the \( \cos(\omega t) \) term in this expression indicates that the force also oscillates as a standing wave. This equation does not appear to be solvable in closed form. However, we can gain some insight into the particle’s behavior by plotting the net force as a function of \( x \) along with the standing pressure wave (see Fig. 6).

![Figure 6: Net force (blue) on a cube in the presence of a standing pressure wave (light red). Note that the force oscillates in time with the same period as the pressure wave.](image)

The force curve in Fig. 6 clearly exhibits regions where the net force on the object is positive, negative, and zero. The places where the force is zero represent equilibrium points, and we can determine their stability by examining the direction of the force on either side. Looking at the solid force curve, we see that the equilibrium points occur at the anti-nodes of the standing pressure wave and they are alternately stable and unstable (stable if the force acts to return a displaced particle to this location, unstable if the force pushes a displaced particle away). However, if we analyze the dashed force curve, which represents the force half a period later, it is apparent that while the equilibrium points are at the same locations, they will all have changed from stable to unstable, or vice versa.

Thus, we see that the equilibrium points occur at pressure anti-nodes and oscillate between stable and unstable at the frequency of the standing wave. In other words, it does not appear as
though the particle will remain at any fixed location. In fact, it is not at all clear what the motion of the particle will be. One thing we can say, however, is that the time average of the force in Eq. (10) is equal to zero. Thus, it would seem that a particle in a standing pressure wave would move, on average, with a constant velocity. On the other hand, the rapidly oscillating force should also result in a small amplitude oscillatory motion. Hence, it may be useful to separate out the effect of the rapidly oscillating force.

2.2 Ponderomotive force analysis

An extension of the analysis from first principles is to consider a particle subject to a rapidly oscillating force. Such a situation is commonly found when a charged particle is subject to a rapidly oscillating electromagnetic field. In this case, the resulting force is called a ponderomotive force [15]. This principle can be used to analyze the motion of an object in a standing wave because the object experiences a force from the rapidly oscillating standing pressure wave.

Following Landau and Lifshitz [16], we will describe the motion of particle as a smooth slow motion \( X(t) \) and a rapid oscillatory motion \( \xi(t) \) so that:

\[
x(t) = X(t) + \xi(t).
\]

(11)

The assumption is that \( \xi(t) \) oscillates rapidly and with a small amplitude compared to \( X(t) \), so that \( X(t) \gg \xi(t) \). The equation of motion of the particle is assumed to have the form

\[
\ddot{x} = g(x) \cos(\omega t).
\]

(12)

Using Eq. (11) and expanding \( g(x) \) about \( X(t) \) to the first order, we find

\[
\dot{X}(t) + \dot{\xi}(t) = [g(X) + \xi g'(X)] \cos(\omega t).
\]

(13)

Because \( \xi(t) \) oscillates much more rapidly than \( X(t) \), we can approximate \( \dot{\xi}(t) \gg \dot{X}(t) \). Moreover, the quantity \( \xi g'(X) \) is a small perturbation on \( g(X) \), so \( g(X) \gg \xi g'(X) \), which gives us

\[
\dot{\xi}(t) \approx g(X) \cos(\omega t).
\]

(14)

Finally, since \( X(t) \) changes much more slowly than \( \xi(t) \), we can treat \( X \) as constant over the time scale of \( \xi \) and integrate twice to get

\[
\xi(t) = -\frac{g(X)}{\omega^2} \cos(\omega t).
\]

(15)

Substituting these results into Eq. (13), we then find that

\[
\dot{X}(t) = \xi g'(X) \cos(\omega t) = -\frac{g(X)g'(X)}{\omega^2} \cos^2(\omega t).
\]

(16)
If we now average over the rapid timescale $2\pi/\omega$, we obtain

$$\langle \ddot{X}(t) \rangle = -\frac{g(X)g'(X)}{2\omega^2},$$

which is equivalent to

$$\langle \ddot{X}(t) \rangle = -\frac{1}{4\omega^2} \frac{d}{dX} [g(X)^2]. \quad (17)$$

We can now use this result to evaluate the long timescale motion of the particle in a standing wave, by noting that Eq. (10) is in the form of Eq. (12), with

$$g(x) = \frac{8p_0a^2}{m} \sin(ka) \sin(kx), \quad (18)$$

from which we can deduce a ponderomotive force given by

$$\langle F(x) \rangle = m\langle \ddot{x} \rangle = -\frac{m}{4\omega^2} \frac{d}{dx} \left\{ \left[ \frac{8p_0a^2}{m} \sin(ka) \sin(kx) \right]^2 \right\}$$

$$= -\frac{16a^4p_0^2\sin^2(ka)}{\omega^2m} k\sin(2kx). \quad (19)$$

Thus, while a simple time average of the force in Eq. (10) leads to zero, here we arrive at an average force that may provide more insight into the phenomenon of acoustic levitation. To understand the implications of Eq. (19), Fig. 7 plots the normalized force along with the standing pressure wave.

![Figure 7: Average force (blue) exerted by a standing pressure wave (light red) on a cube. The average force is independent of time and has a period that is half that of the standing wave.](image)

The graph shows that the average force on the object is zero when the object is at both the nodes and the antinodes. Similar to the previous graph, we can deduce the stability of the
equilibrium points and observe that the stable equilibria occur at the anti-nodes of the pressure wave. This promising result predicts that particles released in a standing pressure wave will end up at pressure anti-nodes. While this analysis provides a glimpse into how a symmetric oscillatory force can give rise to locations of stable equilibrium, it turns out that these locations (pressure anti-nodes) disagree with the research literature on this topic. Hence, to fully understand acoustic levitation requires that we extend the analysis even further.

2.3 A More Detailed Analysis

In the previous two analyses, we considered the force exerted on a particle due to a pressure field alone. But considering pressure alone ignores the fluid aspects of the system. By approaching acoustic levitation from the perspective of fluids in motion, we take into account both the pressure field and the velocity field. Such an analysis will be more thorough (and more complex) than our previous approaches. Let us begin by considering the governing equations of fluid motion. Firstly, the continuity equation, which deals with conservation of mass, is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0,$$

where $\mathbf{u}$ is the velocity of a fluid element and $\rho$ is its density. Secondly, the Navier-Stokes equation, which deals with conservation of momentum, is given by

$$\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p,$$

where $p$ is pressure of the infinitesimal volume. Note that we are ignoring viscous forces, a standard approximation when dealing with air. Normally, we can relate density and pressure using the ideal gas law as we did in Eq. (4), however, in general the equation of state is more complicated. Thus, the final equation is the equation of state that relates the density of a fluid to its pressure is

$$p = p(\rho).$$

Let us consider solving the fluid dynamics problem of a small-amplitude disturbance to an otherwise undisturbed fluid. Following Andrade [17], we begin with a first-order analysis. When the fluid is at rest it has no velocity ($\mathbf{u} = 0$), constant density $\rho_0$ and constant pressure $p_0$. If we introduce a small disturbance, the fields can be represented as a sum of the static field and a first-order perturbation:

$$p = p_0 + p_1
\quad \text{and} \quad
\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1 = \mathbf{u}_1
\quad \text{and} \quad
\rho = \rho_0 + \rho_1,$$

$$p = p_0 + p_1
\quad \text{and} \quad
\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1 = \mathbf{u}_1
\quad \text{and} \quad
\rho = \rho_0 + \rho_1, \quad (23)$$
where we are assuming that $|p_1| \ll p_0$ and $|\rho_1| \ll \rho_0$. Substituting these expressions into Eqs. (20)–(22), we obtain equations that are linear in the small quantities $\rho_1, p_1,$ and $u_1$:

$$\frac{\partial p_1}{\partial t} + \nabla \cdot (\rho_0 u_1) = 0$$  \hspace{1cm} (24)

$$\rho_0 \frac{\partial u_1}{\partial t} = -\nabla p_1$$  \hspace{1cm} (25)

$$p_1 = v_0^2 \rho_1,$$  \hspace{1cm} (26)

where $v_0$ is the speed of sound in the fluid. Combining these linear equations for the fields, we end up with

$$\nabla^2 p_1 = \frac{1}{v_0^2} \frac{\partial^2 p_1}{\partial t^2},$$  \hspace{1cm} (27)

which is the linear wave equation. This equation gives rise to traveling waves and can be solved to find the acoustic pressure as a function of time, which in turn can be used with Eqs. (25) and (26) to find the density and velocity fields. Unfortunately, the first-order analysis just described is not capable of predicting the forces necessary to describe acoustic levitation: as with our first-principles analysis, this leads to an oscillatory force that averages to zero. Therefore, to understand acoustic levitation requires a second-order analysis.

The second-order analysis is significantly more detailed, so we will only discuss the most important features. We begin by including the second order perturbations

$$p = p_0 + p_1 + p_2$$

$$u = u_1 + u_2$$  \hspace{1cm} (28)

$$\rho = \rho_0 + \rho_1 + \rho_2.$$  

Substituting these into Eq. (18), and evaluating the second order terms leads to

$$\nabla p_2 = -\rho_0 \frac{\partial u_2}{\partial t} - \rho_1 \frac{\partial u_1}{\partial t} - \rho_0 (u_1 \cdot \nabla) u_1.$$  \hspace{1cm} (29)

Taking a time average of this equation leads to

$$\nabla \langle p_2 \rangle = -\left( \rho_1 \frac{\partial u_1}{\partial t} \right) - \rho_0 \langle (u_1 \cdot \nabla) u_1 \rangle,$$  \hspace{1cm} (30)

where the first term on the right-hand side of Eq. (29) has averaged to zero. Combining this result with Eqs. (25) and (26), and using the fact that $\nabla (p_1^2) = 2p_1 \nabla p_1$ and $\nabla (u_1 \cdot u_1) = 2(u_1 \cdot \nabla) u_1$, we find
\[
\langle p_2 \rangle = \frac{1}{2 \rho_0 v_0^2} \langle p_1^2 \rangle - \frac{\rho_0}{2} (\mathbf{u}_1 \cdot \mathbf{u}_1) .
\]

This result is the time-averaged radiation pressure. From this expression, the radiation force \( F_{\text{rad}} \) can be calculated by integrating the pressure over the object’s surface,

\[
F_{\text{rad}} = - \int_S \langle p_2 \rangle \mathbf{n} dS ,
\]

where \( \mathbf{n} \) represents the unit normal vector to the surface \( S \).

In general, we need to consider both the incident wave and the scattered wave in the analysis, which complicates the calculation significantly. Gor’kov [18] performed such an analysis for a small rigid sphere of radius \( R \ll \lambda \) in an arbitrary acoustic field, resulting in the radiation potential

\[
U = 2\pi R^3 \left[ \frac{f_1}{3 \rho_0 v_0^2} \left( \langle p_1^{in} \rangle^2 \right) - \frac{f_2 \rho_0}{2} \left( \langle \mathbf{u}_1^{in} \cdot \mathbf{u}_1^{in} \rangle \right) \right] .
\]

from which the force can be calculated as \( \mathbf{F} = -\nabla U \). Notice that although the scattered fields have been taken into account, it is only the incident fields that appear in this expression. The factors \( f_1 \) and \( f_2 \) depend on the mechanical properties of the sphere and fluid, and if the density of the sphere is much larger than the density of air, which is the typical case of interest, then \( f_1 = f_2 = 1 \). Having a means of calculating the radiation force, let us now consider a particle in a standing pressure wave. As previously discussed [see Eq. (6)], the (incident) standing wave is given by

\[
p_1^{in} = p_0 \cos(\omega t) \cos(kx),
\]

where \( p_0 \) is the acoustic pressure amplitude. Then, substituting Eq. (34) into Eq. (25) we obtain the velocity field

\[
\mathbf{u}_1^{in} = \frac{p_0}{\rho_0 v_0} \sin(\omega t) \sin(kx) .
\]

Next, we insert Eqs. (34) and (35) into Eq. (33) to obtain the radiation potential

\[
U = \frac{p_0^2 \pi R^3}{\rho_0 v_0^2} \left[ \frac{\cos^2(kx)}{3} - \frac{\sin^2(kx)}{2} \right] .
\]

Finally, the time-averaged radiation force on a small rigid sphere of radius \( R \) in an acoustic standing wave is found to be

\[
\langle F_{\text{rad}} \rangle = \frac{5\pi R^3 k p_0^2}{6 \rho_0 v_0^2} \sin(2kx) .
\]
It is instructive to compare this expression to the result of our ponderomotive analysis, given by Eq. (19). While both expressions have the same functional dependence, the most obvious difference is that they have opposite signs. This sign difference changes the location of the stable equilibria from pressure anti-nodes (for the ponderomotive force) to pressure nodes, as shown in Fig. 8. To compare the numerical factors out front, we assume $a \ll \lambda$ in Eq. (19), which leads to a ponderomotive force given by

$$\langle F_{\text{pond}}(x) \rangle = -\frac{2a^3kp_0^2}{\rho_m v_0^2} \sin(2kx).$$

Comparing this to Eq. (37), we see that, apart from numerical factors of order unity (likely the result of using a cube instead of a sphere for the ponderomotive analysis), the acoustic radiation force is larger than the ponderomotive force by a factor of $\rho_m/\rho_0$. Since the density of the sphere is much greater than the density of air, the result is that the acoustic radiation force is much greater than the ponderomotive force. This comparison suggests that the ponderomotive analysis misses some important factors that are captured by the more detailed fluid dynamics analysis.

![Figure 8: Acoustic radiation force (blue) exerted by a standing pressure wave (light red) on a cube. As before, the force is independent of time and has a period that is half that of the standing wave. The new force is an inverted form of the force in Fig. 7.](image)

Our initial calculation, when averaged over time, provides a net force of zero. The ponderomotive analysis separates out the fast oscillation and results in an average force that is nonzero. However, it is the full fluid analysis that leads to the result in the research literature. Interestingly, this result was obtained as early as 1934 by King [19].
3 Apparatus

3.1 Schlieren Imaging

In the present experiment, we use a 16.2" diameter concave spherical mirror with a focal length of two meters. We mounted the mirror in an aluminum frame with adjustable feet to level the mirror on uneven surfaces. The light source consists of a 40-W white LED (LED Engin LZC-70WOR) attached to an aluminum heatsink (see Fig. 9a). The LED and heat sink covered by an aluminum tube that contains either a precision pinhole (400-um, Thorlabs P400D) or an adjustable slit (Thorlabs VA100) on its end. Vellum paper is inserted between the LED and the pinhole to ensure a uniform distribution of light. The LED is powered by a Universal LED Controller (ULC), which can be set to output a constant or strobed output. For the strobed output, the ULC can be triggered internally or externally; we triggered it externally with a Tektronix-AFG1022 function generator, which allowed for more precise control.

The wire block is a simple aluminum ring with a thin wire (size 7 piano wire, 0.018" diameter) stretched taut across the center (Fig. 9b). The camera is placed behind the wire block and focused on the object in front of the mirror; it should be positioned such that the lens is also in the same horizontal plane as the wire block, the pinhole, and the center of the mirror. To aid with alignment, we mounted all three objects on an optics track such that the wire block and camera have xz mobility, and the light source has xyz mobility (see Fig. 3). The video camera was a Blackmagic 4K pocket cinema camera with an Olympus M.Zuiko Digital ED 75-300mm f/4.8-6.7 II lens. The camera setup is shown in Fig. 9c.

3.2 Acoustic levitation

The acoustic levitation apparatus consists of a 28-kHz ultrasonic transducer (APC 90-4040) directed at a flat piece of glass suspended horizontally above it by a vertically adjustable clamp (Fig. 10). The transducer was raised to the level of the center of the mirror on a cuboidal piece of
wood for optimum visualization in the schlieren system. The transducer is cylindrically shaped with a conical head and a flat surface at the end, which serves as the driver. The surface of the transducer contains a circular indentation in the center, with a 1-cm diameter, for a flat screw to be attached. When the screw is attached the indentation is effectively 1 mm deep. The objects used for levitation were small Styrofoam balls approximately 3 mm in diameter. The reflector we used was a flat piece of glass with dimensions 6”×7”x0.5” that was attached to a vertically adjustable stage by way of a three-finger clamp.

![Image](image1.jpg)

Figure 10: The acoustic levitation apparatus. Left: The transducer and reflector. Right: Measuring the pressure in front of the mirror.

The function generator used to trigger the ULC had a dual channel output and was used to drive the transducer as well. For successful acoustic levitation, a relatively large amplitude standing wave is required. To accomplish this, we had to ensure maximum output from the transducer, which is achieved by driving it at its resonance frequency. In addition, we amplified the function generator signal using a Servo 120A power amplifier in bridge mode, incorporating a home-made transformer to match impedances between the 50-Ω transducer and the 8-Ω output of the power amplifier.

To measure the pressure changes in the standing wave we used an Earthworks M30 condenser microphone, connected to a Rolls MP13 microphone preamp (which also supplies phantom power to the microphone). The preamp output was directed to a Tektronix TDS1012B oscilloscope for viewing. At 30 kHz, the standing pressure wave has a wavelength of 11.4 mm. Thus, according to the Nyquist-Shannon sampling theorem, mapping out the sound wave requires a spatial resolution smaller than $\lambda/2 = 5.7$ mm. Accordingly, we mounted the microphone on an adjustable stage that had a resolution of 0.001”, and attached a hypodermic needle with a diameter of 1-mm to the head of the microphone. We took pressure measurements of the standing pressure wave in increments of 0.25 mm. The acoustic setup as used with the mirror is shown in Fig. 11.
4 Experimental Results

There are various aspects to this project, so we will present the experimental results in four pieces: (1) white light illumination, (2) density gradients, (3) acoustic levitation, and (4) the combination of both schlieren imaging and acoustic levitation. In principle schlieren imaging can be achieved with any concave mirror, however, there are several advantages to using a large mirror with a long focal-length. Firstly, the larger the mirror the greater the area that will be subject to the schlieren effect. Secondly, the longer the focal length, the greater the distance between the mirror and the camera, which gives deflected light more distance to “bend” around the wire block. This allows the system to detect finer changes in refractive index, effectively increasing the sensitivity of the system.

To maximize the amount of light reaching the camera, the pinhole and wire block should be placed in the horizontal plane of the center of the mirror, as close to the principle axis as possible. Additionally, we increased the amount of light used by replacing the pinhole with an adjustable slit. In order for this to be successful, the slit and the wire block must have the same orientation.
4.1 White light illumination

Our first objective was to ascertain that the system was properly aligned so that schlieren images can be observed. For this purpose, we provided a steady power to the LED. Initial attempts to change the air density in the test area used heat as the source of density changes. Figure 12 shows images obtained using a candle and a hair dryer as the source of heat.

![Figure 12: Schlieren image of a lit candle (left) and a hair dryer (right). The airflow pattern is clearly made visible by the schlieren system.](image)

On the left, density gradients from a lit candle allow us to view the hot air rising from the flame. As expected, the hot air is more dispersed further above the flame as it mixes with the ambient air. On the right, the hairdryer’s exhaust can be seen a few moments after being turned off. This image shows the different velocities of the hot air: faster close to the dryer with more laminar flow, and slower further away from the dryer with more turbulent motion. Notedly, the hot air does not rise much as it exits the dryer but rises after it has decreased in speed. Thus, with this setup it is possible to make use of density differences to observe the behavior of hot air.

The properties of the images can be tuned by adjusting the width of the adjustable slit and the camera aperture. By increasing the slit width to an extent that the image of the slit is wider than the light-block, some of the light from the source “bleeds” around the light-block and into the camera. The result is a bright rectangular strip oriented at an angle corresponding to the angle of the light-block, characterized by streaks of light perpendicular to the strip. An example is shown in Fig. 13a, where the light-block (and slit) were oriented at 45 degrees with respect to the horizontal. It appears that the increased quantity of light ensuing from the slit illuminates even the smallest density changes, but at the cost of a less favorable signal-to-noise ratio.
With an increased slit size, the contrast of the image is controlled by the camera automatically adjusting the aperture of the lens. When the aperture is reduced the image has greater contrast: the features of the brighter spots are visible, but the background looks darker (Fig 13a). The increased sensitivity of the system can be exploited to detect finer density gradients by fixing the camera aperture at its normal size. Thereafter, further increasing the slit width so that the brighter band expands to encompass the mirror allows us to image the slight heat given off by a human hand (Fig. 13b). Note that in both images, the increased quantity of light also highlights edges with a white outline. We believe that this is due to the diffraction of light at the edges of the objects. Ideally, this level of sensitivity should be obtained by increasing the light output, without changing the slit width so that no light “bleeds” around the wire block. Unfortunately, our LED was not capable of such a high output.

4.2 Density gradients

Thus far the Schlieren apparatus can image density gradients but only provides information about the magnitude of the gradient. An improvement to the setup is to add colored filters to the wire light-block, which allows us to determine which way the light is being refracted. Light rays bend toward the normal when passing into a region of higher density and away from the normal when passing into a region of lower density. Therefore, a region of hot air, being less dense, will cause the light rays to diverge and a region of cold air, being more dense, will cause light rays to converge. With the previous setup, light rays would be deflected to one side of the light-block or the other but we would be unable to determine which path they took. However, the addition of color filters on either side of the wire will indicate which direction light is deflected (see Fig. 14).
As an example, consider a setup shown in Fig. 14, which shows the experiment from above. In this situation the wire light block would be oriented vertically and the refracted light will pass to the right or left of the wire (as seen from the camera). Whether light gets deflected to the right or left of the wire is not simply dependent on whether it passes through hot (less dense) air or cold (more dense) air. In fact, as shown in Fig. 14, the direction of deflection depends on the density gradient. As light approaches the left side of a pocket of cold air, it gets bent towards the normal, resulting in the ray being deflected to the right of the light-block and being imaged as red. Alternately, light approaching the right side of the cold pocket is deflected to the left of the light-block and imaged as green. Conversely, for a hot pocket of air, light approaching the left side gets bent away from the normal, will be deflected to the left of the light-block, and imaged as green. Similarly, light approaching the right side of the hot pocket will be deflected to the right of the light-block and imaged as red. Thus, the color scheme of the density changes depends on whether the density is increasing or decreasing (as you move spatially).

Figure 15 shows some schlieren images while using the colored filters. These images show that the addition of color filters provides new information and makes the images more visually striking. In our setup we attached a red filter above the wire and a green filter below the wire.
color filters used were two thin strips of transparent plastic with dimensions 5 cm × 0.2 cm.) In the left image we see clear evidence of the earlier discussion (with left and right now replaced by below and above): the deflected light is green above the ice and red below it, and vice versa for the cup of hot water. Further, one can see pockets of hot air above the cup that display the same color scheme as the cup. We see similar patterns in the image on the right. Although this image shows much more turbulence than the other, the red portions are towards the top of the streams of hot air and the green portions are towards the bottom, as expected.

![Image](image_url)

Figure 15: Schlieren images with colored filters. A cup of hot water and a block of ice (left) and a butane lighter (right). Notice that the hot objects are (predominantly) red on top and green on the bottom while the cold object is green on top and red on the bottom.

Interestingly, we still see white light in the image on the right but not in the image on the left. Light rays passing through these regions experience a density gradient large enough to deflect the light to the extent that it passes above or below the thin color filters and enters the camera as white light. These large density gradients correspond to regions of very high temperature air. Adding another layer of color filters (perhaps blue and orange for good contrast) above and below the existing filters would allow us to capture this information.

### 4.3 Acoustic Levitation

Thus far the filters have been used to obtain information about temperature dependent density changes, but they can also provide information about acoustically induced density changes. As mentioned earlier, acoustic levitation requires a standing wave with sufficiently large acoustic radiation pressure. Hence, it was vital to operate the transducer at its resonance frequency and position the reflector to give rise to a standing wave. Though our transducer was specified to have a resonance frequency of 28 kHz, we were not able to achieve acoustic levitation by driving the
transducer at this frequency even when the power output was maximized. Hence, we decided to measure the resonance frequency of the transducer.

To determine the resonance frequency, we used the function generator’s sweep function to sweep through a range of frequencies while using a measurement microphone and oscilloscope to identify the frequency at which the output has its maximum value (see Fig. 16). After determining the approximate location of the resonance frequency, we stepped through frequencies manually and identified the precise frequency at which the output was maximized. Based on this process, we determined the resonance frequency to be 30.1 kHz.

To ensure a spacing where a standing wave would form we set the reflector to be a half-integer number of wavelengths above the transducer and used a spirit level to ensure it was parallel to the surface of the transducer. We then drove the transducer at resonance and adjust the reflector in minute increments until we established a standing wave. However, before we attempted to achieve acoustic levitation, we needed to be certain that a standing wave had indeed formed, since the wave itself cannot be seen with the naked eye.

There are two main methods for doing this. The first is to use the schlieren system to “see” the wave while adjusting the reflector height. When formed, a standing wave looks like alternating strips of light and dark through the camera. This requires that one walk back and forth between the camera and the acoustic levitation apparatus between adjustments, or to direct the output of the camera to a screen. The second method is to place a few Styrofoam balls on the transducer and adjust the reflector until the balls begin to float just above the surface of the transducer. Using the
second method is more convenient, since the process is contained to the acoustic portion of the experiment.

Figure 17a shows an image of Styrofoam balls being levitated in a standing wave. Notice that the balls are evenly spaced vertically and lie approximately along a straight line. There is clearly an inward radial force that keeps the balls located along the centerline of the transducer (an analysis of this radial force is beyond the scope of this paper). The lowest ball is more displaced than the others and we believe this is caused by the slight indentation in the surface of the transducer.

Figure 17b shows that in addition to a single ball being levitated in the standing wave, multiple balls can also be levitated. In this instance, we observed three balls being levitated together, forming a triangle around the centerline of the transducer. Interestingly, we observed that these balls were slowly rotating about the centerline with a constant angular velocity. Since this rotation continued indefinitely, it suggests that the radiation pressure somehow imparts angular momentum to the balls. Whether this angular momentum is a result of the upward acoustic radiation force—with the three balls acting similarly to a wind turbine—or the radial acoustic radiation force—with slight asymmetries in the shape of the balls giving rise to a preferred direction of torque—is unclear. What is clear is that such a rotational motion is quite unexpected and is something that deserves further exploration.

4.4 Examining Acoustic Levitation

In this section we attempt to experimentally determine whether levitated objects rest at pressure nodes or antinodes. To do this, we take three separate approaches: a visual/geometric comparison, a measurement of the acoustic pressure in the standing wave, and visual approach using our schlieren system.
4.4.1 Geometric Analysis

Our first approach is to examine acoustic levitation based on what we know of the pressure wave. Consider the situation in Fig. 14a in which there are five evenly spaced Styrofoam balls floating between the transducer, driven at 30.1kHz, and the glass reflector. The distance between the transducer and the glass plate is 2.7 cm, which is approximately 2.4 wavelengths. Now, it is well known that when a pressure wave reflects from a hard surface, the result is an anti-node [16]. Thus, if we draw a standing pressure wave having an anti-node at the glass reflector and having two-and-a-half wavelengths, we end up with an anti-node at the surface of the transducer (see Fig. 18).

The schematic diagram of the standing pressure wave demonstrates that the nodes of the waves are evenly spaced from each other, but the top and bottom nodes are located at half that distance from the reflector and transducer. Since the levitated balls are also equally spaced from each other but are only half this distance from the reflector and transducer, we deduce that the balls must be resting at (or, rather, just below) the pressure nodes. (The fact that the lowest ball appears a bit too close to the transducer is a result of the small indentation in the center of the transducer.) Thus, the simplest geometric analysis indicates that the balls rest at pressure nodes.

4.4.2 Pressure measurements

The second method is an attempt to measure the pressure of the standing wave directly. We use a condenser microphone that can measure frequencies from 20 Hz to over 30 kHz, to obtain pressure measurements of the standing wave. As previously described, the microphone is mounted to a vertical translation stage and a 1-mm diameter hypodermic needle is connected to the microphone. The tip of the needle is placed 0.5 cm from the center of the standing wave, and raised using the vertical translation stage. The microphone output is a voltage that is proportional to the pressure above ambient pressure and is analyzed using an oscilloscope. Thus, the peak-to-peak voltage \( V_{pp} \) is a measure of the pressure amplitude at each location.
Figure 19 shows the results of our microphone measurements. These measurements clearly show that the pressure amplitude goes through a series of sinusoidal variations, exactly as one would expect from the standing wave shown in Fig. 18b. It is important to note that the voltage plotted in Fig. 19 is proportional to the amplitude of the standing pressure wave. Thus, the voltage peaks in Fig. 19 correspond to anti-nodes of the standing pressure wave. Also shown in Fig. 19 are the locations of the levitated Styrofoam balls. It is clear that the balls are not resting precisely at the nodes of the standing wave. Instead, they rest just below the nodes. While, this may seem surprising at first, one must remember that the prediction that levitated balls will end up at pressure nodes was made while neglecting gravity. When gravity is taken into account, the equilibrium points will be shifted slightly below the points where the net force exerted by the wave is zero. In other words, the levitated balls should come to rest a little below the pressure nodes, precisely as we have observed.

![Pressure Measurements of Standing Wave](image.png)

Figure 19: Peak-to-peak voltage measurements of a condenser microphone as a function of height above the indentation in the transducer surface. The red data points represent the location of the Styrofoam balls; the black lines represent the transducer (left) and the glass plate (right).

### 4.4.3 Schlieren Analysis

The final method is to implement the Schlieren system to shed some light on acoustic levitation through the identification of density gradients. The Ideal Gas Law, Eq. (4), tells us that pressure is proportional to density (at constant temperature), which means that the pressure gradients in a standing wave are also density gradients. We can use our schlieren apparatus to view the density gradients as well as the directions of increasing and decreasing density of the wave and infer its pressure characteristics. Before we examine a schlieren image of a standing wave, let us first consider an image of a system of known density gradients. To compare the standing wave
with a similar density field, we built a square frame, 8”×8”×0.4”, out of flame-retardant Garolite (G-10/FR4) to withstand high temperatures (see Fig. 20). We drilled holes on the top and bottom, with variable spacing from 5 mm to 19 mm in 1-mm increments, and threaded 26 AWG Nichrome wire through the frame.

![Nichrome wire grid used to compare the standing pressure wave to a known density gradient field.](image)

Passing a current through six of the wires causes them to increase in temperature and heat up the adjacent air. The result is alternating regions of hot and less hot air, with a gradient between them. The wires are arranged vertically so that the rising hot air would not disrupt the pattern of density gradients. Because the density gradients are arranged horizontally, it was necessary to orient the wire light-block vertically as in the schematic earlier (Fig. 14). This orientation ensures that only light that is deflected horizontally will enter the camera, providing information about the horizontal density gradients as shown in Fig. 21.

Examining the image shows that as we approach a region of lower density from the right, we see deflected light as red. Conversely, if we approach the same region from the left, we see deflected light as green. These observations match the basic schematic presented in Fig. 14. Further, if the wires were colder than the surrounding air, we would see the opposite color pattern: green on the right and red on the left. Importantly, both red and green bands represent areas of changing density.
Having established what the bands represent, we can now return to the standing wave and the horizontally oriented light block. By setting the light source to strobe at the same frequency as the pressure wave (30.1 kHz), we can effectively freeze the standing pressure (density) wave at a specific instant of time. Doing so allows us to visualize the wave at the same phase each period in which the colored bands correspond to density (pressure) gradients as shown in Fig. 22.

![Figure 21: A schlieren image of a grid of parallel nichrome wires through which a current is passed to create regions of alternating temperatures.](image1)

Figure 21: A schlieren image of a grid of parallel nichrome wires through which a current is passed to create regions of alternating temperatures.

Having established what the bands represent, we can now return to the standing wave and the horizontally oriented light block. By setting the light source to strobe at the same frequency as the pressure wave (30.1 kHz), we can effectively freeze the standing pressure (density) wave at a specific instant of time. Doing so allows us to visualize the wave at the same phase each period in which the colored bands correspond to density (pressure) gradients as shown in Fig. 22.

![Figure 22: A color schlieren image of a single Styrofoam ball levitating in a standing wave. A mirror image of the levitating ball makes it appear as though there are two balls present.](image2)

Figure 22: A color schlieren image of a single Styrofoam ball levitating in a standing wave. A mirror image of the levitating ball makes it appear as though there are two balls present.
Unlike in the previous color schlieren images with heat, the colors in Fig. 22 are less pronounced when sound is the source of the density changes. This reduced contrast is a result of the density gradients being much smaller in a sound wave than due to temperature changes. Thus, the schlieren system needs to be more sensitive, and more finely tuned to image the standing wave more clearly. Despite the reduced contrast, it is clear that there are both red and green bands within the standing wave. Further, upon careful inspection it is evident that the ball rests in a green band.

The crux of the matter, however, is what this tells us about where the balls rest in the standing wave. Given the example of the wire grid, we know that the colored bands correspond to regions where the density is changing. Hence, a plausible conclusion is that the balls must rest at the antinodes because this is where the density changes the most as the pressure oscillates between the minimum and the maximum. Further, the nodes are points in the wave where the density is stable and does not change in time. However, it is important to realize that the colored bands represent the spatial density gradients at a single specific instant of the wave’s temporal phase. In other words, the density gradients represented by the colored bands are spatial gradients, not temporal gradients. Hence, considering a cosine waveform (as shown schematically in Fig. 18b), the spatial gradients are largest in magnitude at the pressure nodes and zero at the antinodes. The fact that the ball rests in the center of the green band therefore indicates that it rests at (or slightly below) a pressure node. It follows then that if we were to image the same wave, half a period later the ball would be resting in a red band because the density would be changing in the opposite direction. Hence, in the case of multiple levitating balls, they would be resting in both red and green bands. To demonstrate this point, Fig. 23 shows a schlieren image of multiple Styrofoam balls being levitated in both colored bands of a standing wave [11].

Figure 23: A schlieren image of multiple Styrofoam balls levitating in colored bands [11].
5 Conclusion

The phenomenon of acoustic levitation is surprisingly subtle and has generated debate regarding the properties of standing pressure waves. This project revolved around an investigation of acoustic levitation using a schlieren optical system. We began by examining the theory of acoustic levitation and observed that analyses from first principles, both basic and advanced, failed to accurately predict the behavior of a levitating object in a standing pressure wave. Hence, we also considered a fluid analysis of the phenomenon and obtained an accurate prediction of the particle’s behavior.

After establishing the theory, our next step was to achieve schlieren imaging and acoustic levitation. We built a schlieren experiment and an acoustic levitation experiment, and were able to successfully obtain schlieren images and levitate small Styrofoam balls. In addition to white light illumination, we added color filters to the schlieren system to provide information about the imaged density gradients. Finally, we conducted three separate analyses to verify that objects rest just below nodes, as predicted by the research literature. The first analysis was a visual geometric analysis of the standing wave. The second involved measuring the pressure of the standing wave and observing where the balls rested. The final analysis was conducted using schlieren imaging with color filters to provide information about density gradients. Notedly, to our knowledge, the present experiment is the first successful use of schlieren imaging and pressure measurements to verify that objects levitated in acoustic waves come to rest at pressure nodes.

Over the course of the experiment, we noticed several avenues that could motivate further investigation. With respect to the schlieren experiment, the addition of multiple layers of color filters could provide more concrete information about the magnitude of density gradients. Light that is deflected will have different colors depending on whether it travels above or below the wire, and on the magnitude of the deflection it experiences.

With respect to acoustic levitation, we noticed that when multiple balls rested in the same node (see Fig. 17b) they would undergo rotational motion indefinitely, which suggests that the standing pressure wave was imparting angular momentum to the combined structure of the three balls. A future project could involve investigating the mechanism that underlies this phenomenon. Another possible avenue is to examine the effects of multiple transducers and concave reflectors since these variations on the present setup will change the shape of the standing pressure wave. Doing so might allow for the levitation of different sizes and shapes of objects as well.

Optics and acoustics are both intriguing topics that are accessible for an undergraduate physics curriculum. This experiment demonstrates the value of basic optical and acoustic concepts, as well as provides a compelling means of combining the two. Schlieren imaging is a visually
striking experiment and can be used to instruct students in topics such as refraction, oscillations, waves, thermodynamics and basic fluid mechanics.

Specifically, at Dickinson College schlieren imaging and acoustic levitation can be used liberally in Vibrations, Waves and Optics to demonstrate the properties of standing waves and the behavior of light. Schlieren imaging can also be used in all the introductory physics courses to demonstrate thermodynamic concepts. Finally, these concepts are sufficiently complex to motivate and provide avenues for further, in-depth study either in the advanced lab courses or as an independent project. Applications at the senior level would involve the entire process described herein of setting up both experiments, and then combining the two to investigate acoustic levitation.
5 References


[4] Ibid, pp. 5

[5] Ibid, pp. 6–9


[8] Ibid, pp. 18


